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Effects of Suspdended Particles, Magnetic Viscosity and Variable Gravity Field on the Thermosolutal Instability of Couple-Stress Fluid in Porous Medium

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Abstract

In the present paper we investigated the thermosolutal instability of a couple-stress fluid in porous medium including simultaneously the effect of magnetic viscosity, suspended particles and variable gravity field.

Keywords : Suspended Particles, Porous Medium,Rayleigh-Taylor Instability, Magnetic Viscosity, Thermosolutal Instability

Introduction

The problem of thermosolutal instability in fluids in porous medium is of considerable importance in Geophysics, Soil sciences, Ground water hydrology and Astrophysics. Many authors[5-10] have demonstrated the stabilizing influence of magnetic viscosity on thermal convection, thermosolutal convection and gravitational convection. Recently, Sharma and Sharma[11] have studied effect of suspended particles on couple-stress fluid in the presence of rotation and magnetic field. It was found that couple-stress has stabilizing effect and suspended particles have destabilizing effect.

Aim of the Study

The aim of this Paper is to understand the combined effect of suspended particles and rotation on the onset of thermosolutal convection in an elastico-viscous fluid in a porous medium.

Formulation of the Problem and Perturbation Equation

Here we study an infinite, horizontal, incompressible couple-stress fluid layer of thickness d, heated and soluted from below so that, the temperatures, densities and solute concentrations at the bottom surface z = 0 are T_0, ρ_0 and C_0 and at the upper surface z = d are T_d p_d and C_d respectively. This layer is heated and saluted from below such that a uniform temperature gradient β * =|dT/dz|) and uniform solute gradient β*=|dC/dz|) are maintained. The system is acted on by a uniform vertical magnetic field $\vec{H}(0,0,H)$ and variable gravity field $\vec{g}(0,0,-g), g=\lambda g_0, (g_0>0)$ is the value of g at z =0 and λ can be positive or negative as gravity increases or decreases upwards from its value g_0.

Letp, $\rho, \alpha, \alpha', \nu, \mu', g, \mu_e, \eta, T, C, N, \vec{P}$ and $\vec{q}(u, v, w)$ denote, respectively, the pressure, density, thermal coefficient of expansion, and analogous solvent coefficient of expansion, kinematic viscosity, couple-stress viscosity, gravitational acceleration, magnetic permeability, electrical resistivity, temperature, solute concentration, electron number density, stress tensor taking into account the magnetic viscosity and fluid velocity. Then equations expansion the conservation of momentum, mass, temperature, solute mass concentration and equation of state of couple-stress fluid through porous medium are

$$\frac{1}{\varepsilon}\left[\frac{\partial\vec{q}}{\partial t}+\frac{1}{\varepsilon}(\vec{q}.\nabla)\vec{q}\right]=-\frac{1}{\rho_0}\nabla p-\frac{1}{\rho_0}\nabla\vec{P}+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)+\frac{\mu_e}{4\pi\rho_0}(\nabla\times\vec{H})\times\vec{H}+$$

$$\left(\nu-\frac{\mu'}{\rho_o}\nabla^2\right)\nabla^2\vec{q}+\frac{KN}{\varepsilon\rho_o}(\vec{d}_d-\vec{q}) \qquad (2.1)$$

$$\nabla.\vec{q}=0 \qquad (2.2)$$

$$E\frac{\partial}{\partial t}T+\left(\vec{q}.\nabla\right)T+\frac{mN_{C_{pt}}}{\rho_0 c_f}\left[\varepsilon\frac{\partial}{\partial t}+\vec{q}_d.\nabla\right]T=k\nabla^2 T \quad (2.3)$$

$$E'\frac{\partial}{\partial t}T+\left(\vec{q}.\nabla\right)C+\frac{mN'_{C_{pt}}}{\rho_0 c_f}\left[\varepsilon\frac{\partial}{\partial t}+\vec{q}_d.\nabla\right]C=k'\nabla^2 C \quad (2.4)$$

and $$\rho=\rho_0\left[1-\alpha\left(T-T_0\right)+\alpha'\left(C-C_0\right)\right] \quad (2.5)$$

Assuming a uniform particle size, a spherical shape and small relative velocities between the fluid and particles the presence of particles adds an extra force term in the equations of motion (4.2.1), proportional to the velocity difference between the particles and the fluid. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. We assume that the distances between the particles are quite large compared with their diameter so that the interparticle reactions are ignored. The effects of pressure, gravity and Darcian force on the particles are negligibly small and therefore, ignored. Under the above assumptions, if mN is the mass of particles per unit volume, the equations of motion and continuity are

$$mN\left[\frac{\partial\vec{q}_d}{\partial t}+\frac{1}{\varepsilon}\left(\vec{q}_d.\nabla\right)\vec{q}_d\right]=KN\left(\vec{q}-\vec{q}_d\right) \quad (2.6)$$

$$\varepsilon\frac{\partial N}{\partial t}+\left(\nabla.N\vec{q}_d\right)=0 \quad (2.7)$$

Where K = $6\pi\rho\nu\eta'$, η' being particle radius, is the stokes drag coefficient,

$\bar{x}=\left(x,y,z\right), E=\varepsilon\left(1-\varepsilon\right)\left(\frac{\rho_s C_s}{\rho_0 C_f}\right)$ is constant and E' is a constant analogous to E but corresponding to solute rather than heat; k and k' are the thermal diffusivity and solute diffusivity respectively, ρ_s, c_s, ρ_0, c_f denote the density and heat capacity of solid (porous) matrix and fluid, respectively; $\vec{q}_d\left(x,t\right)$ and $N\left(\bar{x},t\right)$ denote filter velocity and number density of the suspended particles, the suffix zero refers to the values at reference level z = 0.

Maxwell's Equations Yield

$$\varepsilon\frac{d\vec{H}}{dt}=\left(\vec{H}.\nabla\right)\vec{q}+\varepsilon\eta\nabla^2\vec{H} \quad (2.8)$$

and $$\nabla.\vec{H}=0 \quad (2.9)$$

where $\frac{d}{dt}=\frac{\partial}{\partial t}+\frac{1}{\varepsilon}\left(\vec{u}.\nabla\right)$ stands for the convection derivative

For the magnetic field along the z-axis, the stress tensor $\vec{P}$ taking into account the magnetic viscosity (Vandakurov[13]) has the components

$$\left.\begin{aligned}
P_{xx}&=-\rho_0\nu_0\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),\\
P_{xy}&=P_{yx}=\rho_0\nu_0\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right),\\
P_{xz}&=P_{zx}=-2\rho_0\nu_0\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right),\\
P_{yz}&=P_{zy}=2\rho_0\nu_0\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right),\\
P_{yy}&=\rho_0\nu_0\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right), P_{zz}=0
\end{aligned}\right\} \quad (2.10)$$

Where $\rho_0\nu_0=\frac{NT}{4\omega_H}$, ω_H being the ion-gyration frequency while N and T are number density and temperature of ions respectively. The steady sate solution is

$$\vec{q}=\left(0,0,0\right),\ \vec{q}_d=\left(0,0,0\right),$$
$$T=-\beta z+T_0, C=-\beta' z+C_0,$$
$$\rho=\rho_0\left(1+\alpha\beta z-\alpha'\beta' z\right), N_0=\text{constant} \quad (2.11)$$

Let

$\theta,\gamma,\delta\rho,\delta p,\vec{q}\left(u,v,w\right),\vec{q}_d\left(l,r,s\right)$ and $\vec{h}\left(h_x,h_y,h_z\right)$

denotes, respectively, the temperature T, solute concentration C, perturbations in density □ , pressure p, fluid velocity (0,0,0), and magnetic field $\vec{H}\left(0,0,H\right)$. The change in density ρ, caused by perturbations θ and γ in temperature and solute concentration is given by

$$\delta\rho=-\rho_0\left(\alpha\theta-\alpha'\gamma\right) \quad (2.12)$$

Then the linearized perturbation equations become

$$\frac{1}{\varepsilon}\frac{\partial\vec{q}}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-\frac{1}{\rho_0}\nabla\vec{p}+\vec{g}\left(\frac{\delta\rho}{\rho_0}\right)+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H}+\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q}$$

$$+\frac{KN_0}{\varepsilon\rho_0}\left(\vec{q}_d-\vec{q}\right) \quad (2.13)$$

$$\nabla.\vec{q}=0 \quad (2.14)$$

$$mN_0\frac{\partial\vec{q}_d}{\partial t}=KN_0\left(\vec{q}-\vec{q}_d\right) \quad (2.15)$$

$$\left(E+b\varepsilon\right)\frac{\partial\theta}{\partial t}=\beta\left(w+bs\right)+k\nabla^2\theta \quad (2.16)$$

$$\left(E'+b'\varepsilon\right)\frac{\partial\gamma}{\partial t}=\beta'\left(w+b's\right)+k'\nabla^2\gamma \quad (2.17)$$

$$\varepsilon\frac{\partial\vec{h}}{\partial t}=\left(\vec{H}.\nabla\right)\vec{q}+\varepsilon\eta\nabla^2\vec{h} \quad (2.18)$$

and $$\nabla.\vec{h}=0 \quad (2.19)$$

where

$$b=\frac{mN_{C_{pt}}}{\rho_0 c_f}, b'=\frac{mN'_{C_{pt}}}{\rho_0 c_f}$$

and w, s are the vertical fluid and particle velocities.

4.3 The Dispersion Relation

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$\left[w,h_z,\theta,\gamma,\zeta,\xi\right]=\left[W(z),K(z),\Theta(z),\Gamma(z),Z(z),X(z)\right]\exp\left[ik_x x+ik_y y+nt\right] \quad (3.1)$$

where k_x and k_y are the wave numbers along x- and y-directions, respectively. $k=\left(k_x^2+k_y^2\right)^{1/2}$ is the resultant wave number and n is the growth rate, which is, in general, a complex constant.

$\zeta = ik_x v - ik_y u$ and $\xi = ik_x h_y - ik_y h_x$ denote, respectively, the z-components of vorticity and current density.

Using equation (2.12) and expression (3.1), equations (2.13) – (2.19) in

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W+\frac{g_0\lambda a^2d^2\left(\alpha\Theta-\alpha'\Gamma\right)}{\nu}$$

$$\left(\frac{\nu_0 d}{\nu}\right)\left(2D^2+a^2\right)DZ-\frac{\mu_e Hd}{4\pi\rho_0\nu}\left(D^2-a^2\right)DK=0 \qquad (3.2)$$

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]Z=\left(\frac{\nu_0}{\nu d}\right)\left(2D^2+a^2\right)DW+\frac{\mu_e Hd}{4\pi\rho_0\nu}DX$$

(3.3)

$$\left(D^2-a^2-E_1p_1\sigma\right)\Theta=-\left(\frac{\beta d^2}{k}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W \qquad (3.4)$$

$$\left(D^2-a^2-E_1'q_1'\sigma\right)\Gamma=-\left(\frac{\beta' d^2}{k'}\right)\left(\frac{B'+\tau_1\sigma}{1+\tau_1\sigma}\right)W \qquad (3.5)$$

$$\left(D^2-a^2-p_2\sigma\right)K=-\left(\frac{Hd}{\eta\varepsilon}\right)DW \qquad (3.6)$$

and

$$\left(D^2-a^2-p_2\sigma\right)X=-\left(\frac{Hd}{\eta\varepsilon}\right)DZ \qquad (3.7)$$

Where we have put a = kd,

$F=\nu'/d^2,\ \sigma=nd^2/\nu,\ x/d=x^*,\ y/d=y^*,$

$z/d=z^*,\ \tau=\frac{m}{k},$

$\tau_1=\frac{\tau\nu}{d^2},\ M=\frac{mN_0}{\rho_0}, B=b+1,\ B'+b'+1,\ E_1=E+b\varepsilon,\ E_1'=E'+b'\varepsilon,$

$F=\frac{\mu'}{\rho_0 d^2\nu},\ D^*=d\frac{d}{dz}=dD$ and

$p_1=\nu/k$ is thermal Prandtl,

$p_2 = \nu/k$ is the magnetic Prandtl number,

$q_1'=\nu/k'$ is the Schmidt number and the superscript* is suppressed.

Eliminating Θ and τ between equations (3.2), (3.4) and (3.5), we obtain on simplification

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W-\frac{g_0\lambda\alpha\beta\alpha^2d^4}{\nu k}\left(\frac{B+\tau_1\alpha}{1+\tau_1\alpha}\right)\frac{W}{\left(D^2-a^2-E_1p_1\sigma\right)}$$

$$+\frac{g_0\lambda a'\beta' a^2d^4}{\nu k'}\left(\frac{B'+\tau_1\sigma}{1+\tau_1\sigma}\right)\frac{W}{\left(D^2-a^2-E_1'q_1'\sigma\right)}+\left(\frac{\nu_0 d}{\nu}\right)\left(2D^2+a^2\right)DZ-\frac{\mu_e HD}{4\pi\rho_0\nu}\left(D^2-a^2\right)DK=0$$

(3.8)

Substituting the value of $R=g_0\alpha\beta d^4/\nu k$ and $S=g_0\alpha'\beta' d^4/\nu k'$ in equation (3.8), we get

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W-R\lambda a^2\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\frac{W}{\left(D^2-a^2-E_1p_1\sigma\right)}$$

$$+S\lambda a^2\left(\frac{B'+\tau_1\sigma}{1+\tau_1\sigma}\right)\frac{W}{\left(D^2-a^2-E_1'q_1'\sigma\right)}+\left(\frac{\nu_0 d}{\nu}\right)\left(2D^2+a^2\right)DZ-\frac{\mu_e Hd}{4\pi\rho_0\nu}\left(D^2-a^2DK=0\right)$$

(3.9)

Substituting the value of K from equation (3.6) in equation (3.9) we get

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W-R\lambda a^2\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\frac{W}{\left(D^2-a^2-E_1p_1\sigma\right)}$$

$$+S\lambda a^2\left(\frac{B'+\tau_1\sigma}{1+\tau_1\sigma}\right)\frac{W}{\left(D^2-a^2-E_1'q_1'\sigma\right)}+\left(\frac{\nu_0 d}{\nu}\right)\left(2D^2+a^2\right)DZ+\frac{\mu_e H^2d^2}{4\pi\rho_0\nu\eta\varepsilon}\ \frac{\left(D^2-a^2\right)D^2W}{\left(D^2-a^2-p_2\sigma\right)}=0$$

(3.10)

Substituting the value of $Q=\mu_e H^2d^2/4\pi\rho_0\nu\eta$ in equation (3.10), we get

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W-R\lambda a^2\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\frac{W}{\left(D^2-a^2-E_1p_1\sigma\right)}$$

$$+S\lambda a^2\left(\frac{B'+\tau_1\sigma}{1+\tau_1\sigma}\right)\frac{W}{\left(D^2-a^2-E_1'q_1'\sigma\right)}+\left(\frac{\nu_0 d}{\nu}\right)\left(2D^2+a^2\right)DZ+\frac{Q}{\varepsilon}\ \frac{\left(D^2-a^2\right)D^2W}{\left(D^2-a^2-p_2\sigma\right)}=0$$

(3.11)

Substituting the value of X from equation (3.7) in equation (3.3), we obtain on simplification

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2+\frac{QD^2}{\varepsilon\left(D^2-a^2-p_2\sigma\right)}\right]Z=\left(\frac{\nu_0}{\nu d}\right)\left(2D^2+a^2\right)DW$$

(3.12)

Substituting the value of Z from equation (3.12) in equation (3.11), we get

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W-R\lambda a^2\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\frac{W}{\left(D^2-a^2-E_1p_1\sigma\right)}$$

$$+S\lambda a^2\left(\frac{B'+\tau_1\sigma}{1+\tau_1\sigma}\right)\frac{W}{\left(D^2-a^2-E_1'q_1'\sigma\right)}+\frac{\left(\frac{\nu_0}{\nu}\right)^2\left(2D^2+a^2\right)^2D^2W}{\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)+\frac{QD^2}{\varepsilon\left(D^2-a^2-p_2\sigma\right)}\right]}$$

$$+\frac{Q}{\varepsilon}\ \frac{\left(D^2-a^2\right)D^2W}{\left(D^2-a^2-p_2\sigma\right)}=0 \qquad (3.13)$$

Substituting the value of $U=\nu_0^2/\nu^2$ in equation (3.13) we obtain on simplification

$$\left\{\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2-E_1p_1\sigma\right)\ \left(D^2-a^2-E_1'q_1'\sigma\right)\right.$$

$$\left(D^2-a^2-p_2\sigma\right)\left(D^2-a^2\right)-R\lambda a^2\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\left(D^2-a^2-E_1'q_1'\sigma\right)\left(D^2-a^2-p_2\sigma\right)+S\lambda\alpha^2\left(\frac{B'+\tau_1\sigma}{1+\tau_1\sigma}\right)$$

$$\left.\left(D^2-a^2-E_1p_1\sigma\right)\left(D^2-a^2-p_2\sigma\right)+\frac{Q}{\varepsilon}\left(D^2-a^2-E_1p_1\sigma\right)\ \left(D^2-a^2-E_1'q_1'\sigma\right)\ \left(D^2-a^2\right)D^2\right\}$$

$$\left\{\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)+\frac{QD^2}{\varepsilon\left(D^2-a^2-p_2\sigma\right)}\right]\right.W+U\left(2D^2+a^2\right)^2$$

$$\left(D^2-a^2-E_1q_1\sigma\right)\ \left(D^2-a^2-p_2\sigma\right)D^2W=0 \qquad (3.14)$$

Here also we consider the case where both boundaries are free as well as perfect conductors of heat and solute concentration, while the adjoining medium is perfectly conducting. The appropriate boundary conditions, with respect to which equations (3.2) – (3.7) must be solved are (Chandrasekhar[1])

$$W=D^2W=X=DZ=0,\ \ \Theta=0,\ \Gamma=0,\ \text{at } z=0 \text{ and } 1$$

K = 0 on perfectly conducting boundary and h_x, h_y, h_z are continuous.

The case of two free boundaries though little artificial, is the most appropriate for stellar atmospheres (Spiegel[12]). Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for z =0 and z =1 and hence proper solution of equation (3.14) characterizing the lowest mode is

$W = W_0 \sin \pi z$

where W_0 is constant. Substituting (3.16) in (3.14) and letting $R_1 = R/\pi^4$,

$S_1 = S/\pi^4,\ Q_1 = Q/\pi^2,\ x = a^2/\pi^2,\ i\sigma_1 = \sigma/\pi^2$ and $F_1 = \pi^2 F$, we obtain the dispersion

Relation

$$R_1\lambda x = \left(\frac{1+i\sigma_1\tau_1\pi^2}{B+i\sigma_1\tau_1\pi^2}\right)\left[\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{i\sigma_1\tau_1\pi^2+1}\right)+F_1(1+x)^2+(1+x)\right](1+x)(1+x+E_1p_1i\sigma_1)$$

$$+\frac{S_1\lambda x\left(B'+i\sigma_1\tau_1\pi^2\right)\ \left(1+x+E_1p_1i\sigma_1\right)}{\left(B+i\sigma_1\tau_1\pi^2\right)\ \left(1+x+E_1'q_1'i\sigma_1\right)}+$$

$$\left(\frac{1+i\sigma_1\tau_1\pi^2}{B+i\sigma_1\tau_1\pi^2}\right)\left[\frac{U(x-2)^2(1+x+E_1p_1i\sigma_1)(1+x+p_2i\sigma_1)}{(1+x+p_2i\sigma_1)\left\{\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{i\sigma_1\tau_1\pi^2+1}\right)+F_1(1+x)^2+(1+x)\right\}+\frac{Q_1}{\varepsilon}}\right]$$

$$+\left(\frac{1+i\sigma_1\tau_1\pi^2}{B+i\sigma_1\tau_1\pi^2}\right)\left[\frac{Q_1(1+x)(1+x+E_1p_1i\sigma_1)}{\varepsilon(1+x+p_2i\sigma_1)}\right] \qquad (3.17)$$

Equation (3.17) is the required dispersion relation studying the effects of of magnetic field, stable solute gradient, magnetic viscosity, varying gravity field and suspended suspended particles on thermosolutal instability of couple-stress fluid in the presence of vertical magnetic field in porous medium

Stability of the System and Oscillatiory Modes

In this section, we consider the possibility of oscillatory modes, if any, on the couple-stress fluid due to the presence of magnetic field, suspended particles, stable solute gradient, magnetic viscosity and varying gravity field.

Multiplying equation (3.2) by W*, the complex conjugate of W and making use of equations (3.4) and (3.5), we obtain on simplification

$$\left[\frac{\alpha}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)WW^*-\frac{g_0\lambda ka^2}{\nu\beta}\left(\frac{1+\tau_1\sigma}{B+\tau_1\sigma}\right)$$

$$\left(D^2-a^2-E_1p_1\sigma\right)\Theta^2+\frac{g_0\lambda\alpha'k'a^2}{\nu\beta'}\left(\frac{1+\tau_1\sigma}{B'+\tau_1\sigma}\right)\left(D^2-a^2-E_1'q_1'\sigma\right)\Gamma^2+\left(\frac{\nu_0 d}{\nu}\right)$$

$$\left(2D^2+a^2\right)DZW^*-\frac{\mu_e Hd}{4\pi\rho_0\nu}\left(D^2-a^2\right)DKW''=0) \qquad (4.1)$$

Solving the equations (4.1) and (3.3), we have

$$\left[\frac{\alpha}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)WW^*-\frac{g_0\lambda ka^2}{\nu\beta}\left(\frac{1+\tau_1\sigma}{B+\tau_1\sigma}\right)$$

$$\left(D^2-a^2-E_1p_1\sigma\right)\Theta^2+\frac{g_0\lambda\alpha'k'a^2}{\nu\beta'}\left(\frac{1+\tau_1\sigma}{B'+\tau_1\sigma}\right)\left(D^2-a^2-E_1'q_1'\sigma\right)\Gamma^2+\left(\frac{\nu_0 d}{\nu}\right)$$

$$\left(2D^2+a^2\right)Z^2-\frac{\mu_e Hd^3}{4\pi\rho_0\nu}DXZ-\frac{\mu_e Hd}{4\pi\rho_0\nu}\left(D^2-a^2\right)DKW'=0 \qquad (4.2)$$

Making use of equation (4.2) and (3.6), we get

$$\left[\frac{\alpha}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)WW^*-\frac{g_0\lambda ka^2}{\nu\beta}\left(\frac{1+\tau_1\sigma}{B+\tau_1\sigma}\right)$$

$$\left(D^2-a^2-E_1p_1\sigma\right)\Theta^2+\frac{g_0\lambda\alpha'k'a^2}{\nu\beta'}\left(\frac{1+\tau_1\sigma}{B'+\tau_1\sigma}\right)\left(D^2-a^2-E_1'q_1'\sigma\right)\Gamma^2+\left(\frac{\nu_0 d}{\nu}\right)$$

$$\left(2D^2+a^2\right)Z^2-\frac{\mu_e Hd^3}{4\pi\rho_0\nu}DXZ+\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}\left(D^2-a^2\right)\ \left(D^2-a^2-p_2\alpha\right)K^2=0 \qquad (4.3)$$

Solving the equations (4.3) and (3.7), integrating over the range of z and making use of the boundary conditions (3.15), we obtain on simplification

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)\right]\int_0^1\left(|DW|^2+a^2|W|^2\right)dz+F\int_0^1\left(|D^3W|^2+3a^2|D^2W|^2+a^6|W|^2\right)dz$$

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)\right]\int_0^1\left(|DW|^2+a^2|W|^2\right)dz+F\int_0^1\left(|D^3W|^2+3a^2|D^2W|^2+a^6|W|^2\right)dz$$

$$dz+\frac{g_0\lambda\alpha'k'a^2}{\nu\beta'}\left(\frac{1+\tau_1\sigma}{B'+\tau_1\sigma}\right)\int_0^1\left[\left(|D\Gamma|^2+a^2|\Gamma|^2\right)+E_1'q_1'\sigma\left(|\Gamma|^2\right)\right]dz+\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}$$

$$\int_0^1\left(|D^2K|^2+2a^2|DK|^2+a^4|K|^2\right)+\left(|DK|^2+a^2|K|^2\right)dz+d^2\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)\right]$$

$$\int_0^1\left(|Z|^2\right)dz+Fd^2\int_0^1\left(|D^2Z|^2+2a^2|DZ|^2+a^4|Z|^2\right)dz+d^2\int_0^1\left(|DZ|^2+a^2|Z|^2\right)dz+\frac{\mu_e\eta\varepsilon d^2}{4\pi\rho_0\nu}$$

$$\int_0^1\left[\left(|DX|^2+a^2|X|^2\right)+p_2\sigma\left(|X|^2\right)\right]dz \qquad (4.4)$$

Rewriting equation (4.4) in the form

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)\right]I_1+FI_2+I_3-\frac{g_0\lambda\alpha ka^2}{\nu\beta}\left(\frac{1+\tau_1\sigma}{B+\tau_1\sigma}\right)\left[I_4+E_1p_1\sigma^*I_5\right]+$$

$$\frac{g_0\lambda a'k'a^2}{\nu\beta'}\left(\frac{1+\tau_1\sigma}{\beta'+\tau_1\sigma}\right)\left[I_6+E_1'q_1'\sigma^*I_7\right]+\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}\left[I_8+p_2\sigma^*I_9\right]+$$

$$d^2\left[\frac{\sigma^*}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma^*+1}\right)\right]I_{10}+Fd^2I_{11}+d^2I_{12}+\frac{\mu_e\eta\varepsilon d^2}{4\pi\rho_0\nu}\left[I_{13}+p_2\sigma I_{14}\right]=0 \qquad (4.5)$$

Where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,$$

$$I_2=\int_0^1\left(|D^3W|^2+3a^4|DW|^2+3a^2|D^2W|^2+a^6|W|^2\right)dz,$$

$$I_3=\int_0^1\left(|D^2W|^2+a^4|W|^2+2a^2|DW|^2\right)dz,$$

$$I_4=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz, \qquad I_5=\int_0^1\left(|\Theta|^2\right)dz,$$

$$I_6=\int_0^1\left(|D\Gamma|^2+a^2|\Gamma|^2\right)dz, \qquad I_7=\int_0^1\left(|\Gamma|^2\right)dz,$$

$$I_8=\int_0^1\left(|D^2K|^2+a^4|K|^2+2a^2|DK|^2\right)dz, \qquad I_9=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,$$

$$I_{10}=\int_0^1\left(|Z|^2\right)dz, \qquad I_{11}=\int_0^1\left(|D^2Z|^2+a^4|Z|^2+2a^2|DZ|^2\right)dz,$$

$$I_{12}=\int_0^1\left(|DZ|^2+a^2|Z|^2\right)dz, \qquad I_{13}=\int_0^1\left(|DK|^2+a^2|X|^2\right)dz,$$

$$I_{14}=\int_0^1\left(|X|^2\right)dz=0 \qquad (4.6)$$

The integrals $I_1 - I_{14}$ are all positive definite. Equation (4.5) can be written as

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{\tau_1\sigma+1}\right)\right]I_1+FI_2+I_3-\frac{g_0\lambda\alpha ka^2}{\nu\beta}\left(\frac{1+\tau_1\sigma}{B+\tau_1\sigma}\right)\left[I_4-E_1p_1\sigma I_5\right]+\frac{g_0\lambda\alpha'k'a^2}{\nu\beta'}$$

$$\left(\frac{1+\tau_1\sigma}{B'+\tau_1\sigma}\right)\left[I_6-E_1'q_1'\sigma I_7\right]+\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}\left[I_8-p_2\sigma I_9\right]+d^2\left[-\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1-\tau_1\sigma}\right)\right]I_{10}+Fd^2I_{11}$$

$$+d^2I_{12}+\frac{\mu_3\eta\varepsilon d^2}{4\pi\rho_0\nu}\left[I_{13}+p_2\sigma I_{14}\right]=0 \qquad (4.7)$$

Substituting $\sigma = i\sigma_0$, where σ_0 is real, in equation (4.7), we obtain on simplification

$$\left[\frac{i\sigma_0}{\varepsilon}\left\{1+\frac{M}{1+(\tau_1\sigma_0)^2}-\frac{M\tau_1 i\sigma_0}{1+(\tau_1\sigma_0)^2}\right\}\right]I_1+FI_2+I_3-\frac{g_0\lambda\alpha ka^2}{\nu\beta}\left[\frac{\tau_1 i\sigma_0(B-1)}{B^2+(\tau_1\sigma_0)^2}+\frac{B+(\tau_1\sigma_0)^2}{B^2+(\tau_1\sigma_0)^2}\right]$$

$$\left[I_4-E_1p_1i\sigma_0I_5\right]+\frac{g_0\lambda\alpha'k'a^2}{\nu\beta'}\left[\frac{\tau_1 i\sigma_0(B'-1)}{B'^2+(\tau_1\sigma_0)^2}+\frac{B'+(\tau_1\sigma_0)^2}{B'^2+(\tau_1\sigma_0)^2}\right]\left[I_6-E_1'q_1'i\sigma_0I_7\right]+\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}$$

$$\left[I_8-p_2i\sigma_0I_9\right]-d^2\left[\frac{i\sigma_0}{\varepsilon}\left\{1+\frac{M}{1+(\tau_1\sigma_0)^2}+\frac{M\tau_1 i\sigma_0}{1+(\tau_1\sigma_0)^2}\right\}\right]I_{10}+Fd^2I_{11}+d^2I_{12}+\frac{\mu_e\eta\varepsilon d^2}{4\pi\rho_0\nu}$$

$$\left[I_{13}+p_2i\sigma_0I_{14}\right]=0 \qquad (4.8)$$

Equating the imaginary parts of equation (4.8), we obtain

$$\sigma_0\left[\frac{1+M+(\tau_1\sigma_0)^2}{\varepsilon\{1+(\tau_1\sigma_0)^2\}}(I_1-d^2I_{10})-\frac{g_0\lambda\alpha ka^2}{\nu\beta}\left\{\frac{\tau_1(B-1)}{B^2+(\tau_1\sigma_0)^2}I_4-E_1p_1\frac{B+(\tau_1\sigma_0)^2}{B^2+(\tau_1\sigma_0)^2}I_5\right\}\right.$$
$$+\frac{g_0\lambda\alpha'k'a^2}{\nu\beta'}\left\{\frac{\tau_1(B'-1)}{B'^2+(\tau_1\sigma_0)^2}I_6-E'_1q'_1\frac{B'+(\tau_1\sigma_0)^2}{B'^2+(\tau_1\sigma_0)^2}I_7\right\}-\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}p_2I_9+$$
$$\left.\frac{\mu_e\eta\varepsilon d^2}{4\pi\rho_0\nu}p_2I_{14}\right]=0 \qquad (4.9)$$

Equation (4.9) implies that $\sigma_0=0 \text{ or } \sigma_0\neq 0$ which means that modes may be non-oscillatory or oscillatory. In the absence of suspended particles, magnetic field, magnetic viscosity, stable solute gradient, varying gravity field and couple-stress, equation (4.9) reduces to

$$\sigma_0\left(\frac{1_1}{\varepsilon}+\frac{g_0\lambda\alpha ka^2}{\nu\beta}E_1p_1I_5\right)=0 \qquad (4.10)$$

and terms in bracket are positive definite. Thus $\sigma_0 = 0$, which means the modes are non-oscillatory and the principle of exchange of stabilities is satisfied for a porous medium in the absence of suspended particles, magnetic field, magnetic viscosity, stable solute gradient, varying gravity field and couple-stress. The oscillatory modes are introduced due to the presence of suspended particles, magnetic viscosity, stable solute gradient, varying gravity field, magnetic field and couple-stress which were non-existent in their absence.

The Stationary Convection

Equation (4.3.17) for stationary convection (i.e. σ= 0) reduces to

$$R_1=\frac{1}{B}\left[\frac{F_1(1+x)^4+(1+x)^3}{\lambda x}+S_1B'+\frac{U(x-2)^2(1+x)^2}{\lambda x\left\{F_1(1+x)^3+x(1+x)^2+\frac{Q_1}{\varepsilon}\right\}}+\frac{Q_1(1+x)}{\varepsilon\lambda x}\right]$$

which expresses the modified Rayleigh number R_l as a function of the dimensionless wave number x and the parameters Q_1, U, S_1, B and F_1.

To study the effects of suspended particles, stable solute gradient, magnetic viscosity, magnetic field and couple-stress on R_1; we examine the behaviour of $\frac{dR_1}{dB},\frac{dR_1}{dS_1},\frac{dR_1}{dU},\frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dF_1}$ respectively. Equation (5.1) yields.

$$\frac{dR_1}{dB}=-\frac{1}{B^2}\left[\frac{1+x}{\lambda x}\left\{F_1(1+x)^3+(1+x)^2+\frac{U(x-2)^2(1+x)}{F_1(1+x)^3+(1+x)^2+\frac{Q_1}{\varepsilon}}+\frac{Q_1}{\varepsilon}\right\}+S_1B'\right] \qquad (5.2)$$

which is negative implying thereby that the effect of suspended particles is to destabilize the system when gravity increases upwards from its value g_0 and stabilizes the system when gravity decreases upwards, if

$$\frac{1+x}{\lambda x}\left\{F_1(1+x)^3+(1+x)^2+\frac{U(x-2)^2(1+x)}{F_1(1+x)^3+(1+x)^2+\frac{Q_1}{\varepsilon}}+\frac{Q_1}{\varepsilon}\right\}>S_1B'$$

For equation (4.5.1), we get

$$\frac{dR_1}{dS_1}=\frac{B'}{B} \qquad (5.3)$$

Equation (5.3) show that stable solute gradient has stabilizing effect

$$\frac{dR_1}{dU}=\frac{(x-2)^2(1+x)^2}{B\lambda x\left\{F_1(1+x)^3+(1+x)^2+\frac{Q_1}{\varepsilon}\right\}} \qquad (5.4)$$

From equation (4.5.4), we see that magnetic viscosity has stabilizing effect on the system in porous medium as gravity increases upward from its value g_0. It is evident from equation (5.1) that

$$\frac{dR_1}{dQ_1}=\frac{(1+x)}{B\lambda x\varepsilon^2}\left[\varepsilon F_1^2(1+x)^6+2\varepsilon F_1(1+x)^5+\varepsilon(1+x)^4+2Q_1F_1(1+x)^3+2Q_1(1+x)^2\right.$$
$$\left.+\frac{Q_1^2}{\varepsilon}-\varepsilon U(x-2)^2(1+x)\right]\times\left[F_1(1+x)^3+(1+x)^2+\frac{Q_1}{\varepsilon}\right]^{-2} \qquad (5.5)$$

which implies that magnetic field stabilizes the system when gravity is increasing upwards i.e. (λ>0) and destabilizes the system when gravity is decreasing upwards.

Also from equation (5.1), we get

$$\frac{dR_1}{dF_1}=\frac{(1+x)^4}{\lambda xB}\left[F_1^2(1+x)^6\varepsilon^2+2Q_1F_1(1+x)^3\varepsilon^2+2F_1(1+x)^5\varepsilon^2+(1+x)^4\varepsilon^2+2Q_1(1+x)^2\varepsilon\right.$$
$$\left.+Q_1^2-U(x-2)^2(1+x)\varepsilon^2\right]\times\left[F_1\varepsilon(1+x)^3+\varepsilon(1+x)^2+Q_1\right]^2 \qquad (5.6)$$

Result and Discussion

Equation (5.6) show that couple-stress has stabilizing or destabilizing effects on thermosolutal instability as gravity decreases or increases upwards.

The dispersion relation (5.1) is analysed numerically. Graph have been plotted by given some numerical values to the parameters, to depict the stability characteristics. In Fig. 1, R_1 is plotted against. B for f_1 = 0.6, S_1 = 7, ε= 0.6, λ = 1, U = 20, Q_1 = 15 and B' = 2 for fixed wave numbers x = 0.4 and x = 0.7. For the wave numbers x = 0.4 and x = 0.7, suspended particles have a destabilizing effect. In figures 2, R_1 is plotted against S_1 for F_1 = 0.6, λ= 0.6, Q_1= 1, U = 20, Q_1 = 15, B' = 2 and B = 3 for fixed wave numbers x = 0.4 and x = 0.7. This shows that stable solute gradient has a stabilizing effect. In figures 3, R_1 is plotted against U for F_1 – 0.6, ε= 0.6, λ = 1, Q_1=15, B' = 2, B = 3 and S_1 = 7 for fixed wave numbers x = 0.4 and x =0.7. The Rayleigh number increases with increase in magnetic viscosity parameter showing its stabilizing effects on the thermosolutal instability. In figures 4, R_1 is plotted against Q_1 for F_1 = 0.6, ε.

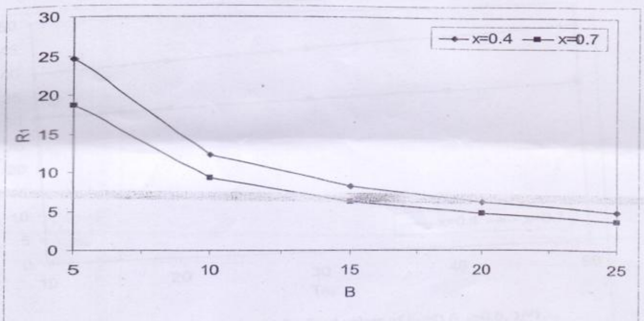

Fig. 1. R_1 is plotted against B for fixed values of F_1=0.6, S_1=7, ε=0.6, λ=1, U=20, Q_1=15, B'=2 and wave number x =0.4 and x =0.7.

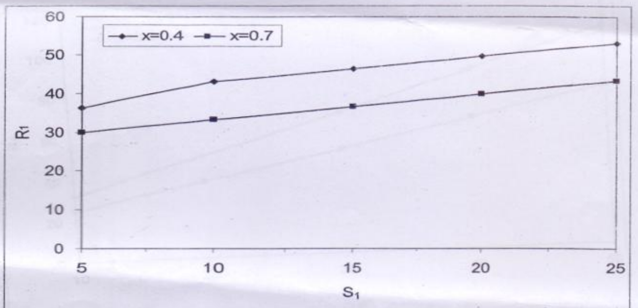

Fig. 2. R_1 is plotted against S_1 for fixed values of F_1=0.6, ε=0.6, λ=1, U=20, Q_1=15, B'=2, B = 3 and wave number x =0.4 and x =0.7.

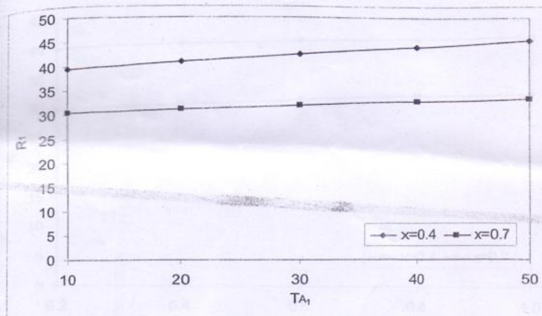

Fig. 3. R_1 is plotted against U for fixed values of F_1=0.6, ε=0.6, λ=1, Q_1=15, B'=2, B=3, S_1=7 and wave number x =0.4 and x =0.7.

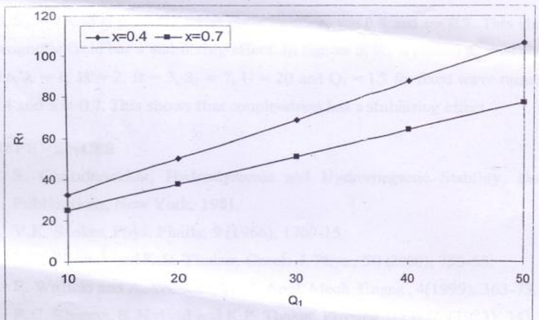

Fig. 4. R_1 is plotted against Q_1 for fixed values of F_1=0.6, ε=0.6, λ=1, B'=2, B=3, S_1=7, U=20 and wave number x =0.4 and x =0.7.

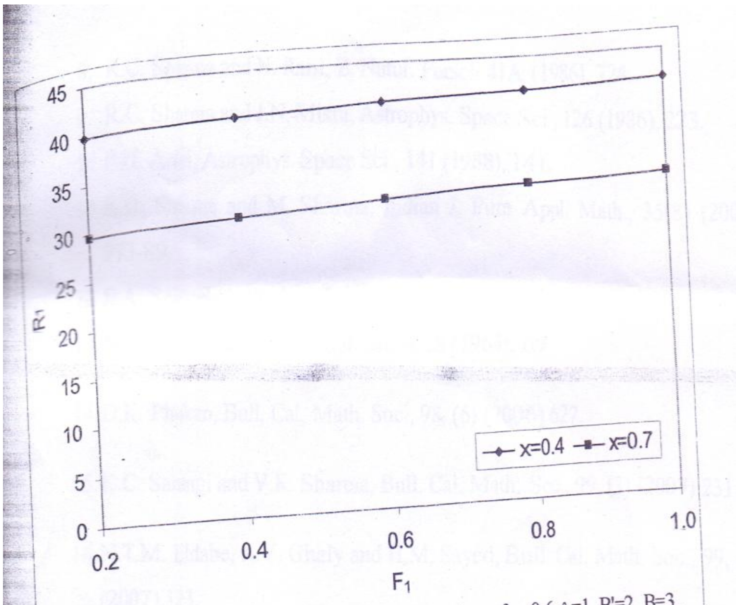

Fig. 5. R_1 is plotted against F_1 for fixed values of ε=0.6, λ=1, B'=2, B=3, S_1=7, U=20, Q_1=15 and wave number x=0.4 and x=0.7.

= 0.6, λ = 1, B' = 2, B =3, S_1 = 7, and U = 20 for fixed wave numbers x = 0.4 and x =0.7. This shows that magnetic field has a stabilizing effect. In figures 5, R_1 is plotted against F_1 for ε = 0.6, λ = 1, B' = 2, B = 3, S_1 = 7, U = 20 and Q_1 = 15 for fixed wave numbers x = 0.4 and x = 0.7. This shows that couple-stress has a stabilizing effect.

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